CSTR: Contaminant in and moving through total pore space (No η_e)

$$\frac{d(CV_T\,\eta)}{dt}+\frac{d(qV_T\,\beta_d)}{dt}=QC_o-QC-V_T\eta\lambda C-V_T\beta_d\lambda q \tag{1}$$

$$\textit{Where}\quad Q=V_p\eta A_x \tag{2}$$

$$\tau=\frac{V_T\theta}{Q}=\frac{V_T}{V_pA_x} \tag{3}$$

$$q=CK_d \tag{4}$$

$$(V_T\eta+V_T\beta_dK_d)\frac{dC}{dt}=QC_o-[Q+\lambda(V_T\eta+V_T\beta_dK_d)]C \tag{5}$$

Divide through by $V_T\eta$

$$(1+\frac{\beta_dK_d}{\eta})\frac{dC}{dt}=\frac{QC_o}{V_T\eta}-[\frac{Q}{V_T\eta}+\lambda(1+\frac{\beta_dK_d}{\eta})]C \tag{6}$$

Let

$$\boxed{R_f=1+\frac{\beta_dK_d}{\eta} \qquad \textbf{(9)}}$$

and Divide through by R_f ∴

$$\frac{dC}{dt}=\frac{C_o}{R_f\tau}-[\frac{1}{R_f\tau}+\lambda]C \tag{10}$$

$$\boxed{\frac{dC}{dt}=\frac{C_o}{R_f\tau}-(\frac{1+R_f\tau\lambda}{R_f\tau})C \qquad (11)}$$

Let $A=\frac{C_o}{R_f\tau}$ (12)

$B=(\frac{1+R_f\tau\lambda}{R_f\tau})$ (13)

Substituting:

$$\frac{dC}{dt}=A-BC \quad (14)$$

Rearrange:

$$\frac{dC}{(A-BC)}=dt \quad (15)$$

Let $u=A-BC$ (16)

$du=-BdC$ (17)

$$(-\frac{1}{B})\int_{u_1}^{u_2}\frac{du}{u}=\int_{t_1}^{t_2}dt \quad (18)$$

$$(-\frac{1}{B})\ln(u)\Big|_{u_1}^{u_2}=(t_2-t_1) \quad \mathbf{(19)}$$

$$(-\frac{1}{B})\ln(A-BC)\Big|_{c_1}^{c_2}=t_2-t_1 \quad (20)$$

where at t=t, c=c

$t=t_2$, $c=c_2$

Note that at $t=t_1=0$; $c=c_1=c_i$

$$(-\frac{1}{B})\ln[\frac{A-BC_2}{A-BC_1}]=(t_2-t_1) \tag{21}$$

$$\frac{A-BC_2}{A-BC_1}=\exp(-B(t_2-t_1)) \tag{22}$$

$$A-BC_2=(A-BC_1)\ \exp(-B(t_2-t_1)) \tag{23}$$

$$C_2=\frac{1}{B}[A-[(A-BC_1)\ \exp(-B(t_2-t_1))]] \tag{24}$$

$$\boxed{C_2=\frac{A}{B}-[(\frac{A}{B}-C_1)\ \exp(-B(t_2-t_1))] \qquad (25)}$$

$$When\quad A=\frac{C_o}{R_f\tau} \tag{26}$$

$$B=(\frac{1+R_f\tau\lambda}{R_f\tau})$$

$$\boxed{C_2=(\frac{C_0}{1+R_f\tau\lambda})-[[(\frac{C_o}{1+R_f\tau\lambda})-C_1]\ \exp(-(\frac{1+R_f\tau\lambda}{R_f\tau})(t_2-t_1))] \qquad (27)}$$

When $c_o=0$

$t_1=0$

$c_1=c_i$

$$C_2=(C_i)\ \exp[-(\frac{1+R_f\tau\lambda}{R_f\tau})(t_2)] \tag{28}$$